

Quantum Clock Synchronisation

Richard Torsa (Univ. of Bristol UK)

Daniel Abrams
Jonathan Dowling
Colin Williams } (JPL, Caltech)

PRL vol 85, 28 August 00, p 2010-2013

Quantum Clock

Any time evolving 2-level quantum system

Stationary states:

$$|0\rangle \rightarrow e^{i\omega t/\hbar} |0\rangle \quad \boxed{\omega = \frac{E_1 - E_0}{\hbar}}$$

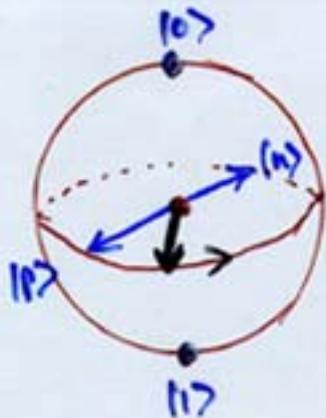
$$|1\rangle \rightarrow e^{i\omega t/\hbar} |1\rangle$$

e.g.

- spin $\frac{1}{2}$ particle in \pm -magnetic field $|0\rangle = |-\frac{1}{2}\rangle$ $|1\rangle = |\frac{1}{2}\rangle$
- Cs atom $|0\rangle, |1\rangle$ are two energy levels

Non-stationary state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \rightsquigarrow |\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-i\frac{\omega t}{2}} |0\rangle + e^{i\frac{\omega t}{2}} |1\rangle \right)$$



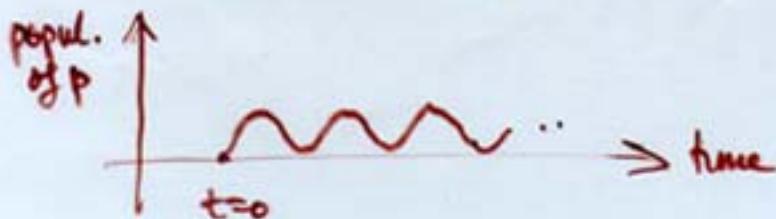
$|\Psi(t)\rangle$ rotates around equator of Bloch sphere, period $\frac{2\pi}{\omega}$.

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \quad |n\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$$

$$\text{So } |\Psi(t)\rangle = \frac{1}{\sqrt{2}} \left(\cos \frac{\omega t}{2} |0\rangle + \sin \frac{\omega t}{2} |1\rangle \right)$$

$$\text{Prob}(P) = \frac{1}{2} (1 + \cos \omega t)$$

Monitor population of P in an ensemble



Oscillation of population
= "ticking of clock"

A subtle point - choice of "frame of reference" in state space.

Given physical states $|0\rangle$ & $|1\rangle$ prescribed,

the state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and operations like Hadamard H are not yet uniquely determined.

- a choice of phase remains.

spin $\frac{1}{2}$ in $\pm z$ magnetic field

$|0\rangle$ & $|1\rangle$ are $(\pm \hat{z})$ spin states (defined by field direction)

- $|+\rangle = |0\rangle + |1\rangle$ determined by choice of an x direction in plane \perp to z .
- measure in $|0\rangle \pm |1\rangle$ basis $\equiv x$ -spin measurement.

Cs atom

$|0\rangle$ & $|1\rangle$ are selected energy levels (known for all Cs atoms)

- H operation : determined by a laser pulse with some chosen phase.
- measure in $|0\rangle \pm |1\rangle$ basis : apply H and measure 0/1 basis.

In general on Bloch sphere :

- choice of $|0\rangle + |1\rangle \Leftrightarrow$ any point on the equator.
- there are unitary transfs leaving physical states $|0\rangle, |1\rangle$ fixed, but moving the points on equator.

If a protocol uses information of $|0\rangle + |1\rangle$ or H choice

- more than once , or
- if more than one party needs to use the information

then the choice must be consistent.



two uses:

Cs atom: phase locked laser

spin $\frac{1}{2}$ in mag. field: ||| \parallel \times directions \perp to z .

For a distributed protocol:

Cs atom: Alice & Bob cannot apply same H without phase locked lasers
⇒ syntonicity already.

spin $\frac{1}{2}$ in mag. field: A & B (given z direction)
need to use ||| \parallel \times directions.

vectors in \mathbb{K} vs rays in \mathbb{K} :

Given vectors $|0\rangle, |1\rangle \Rightarrow$ all other vectors $x_0|0\rangle + x_1|1\rangle$ have unique meaning.

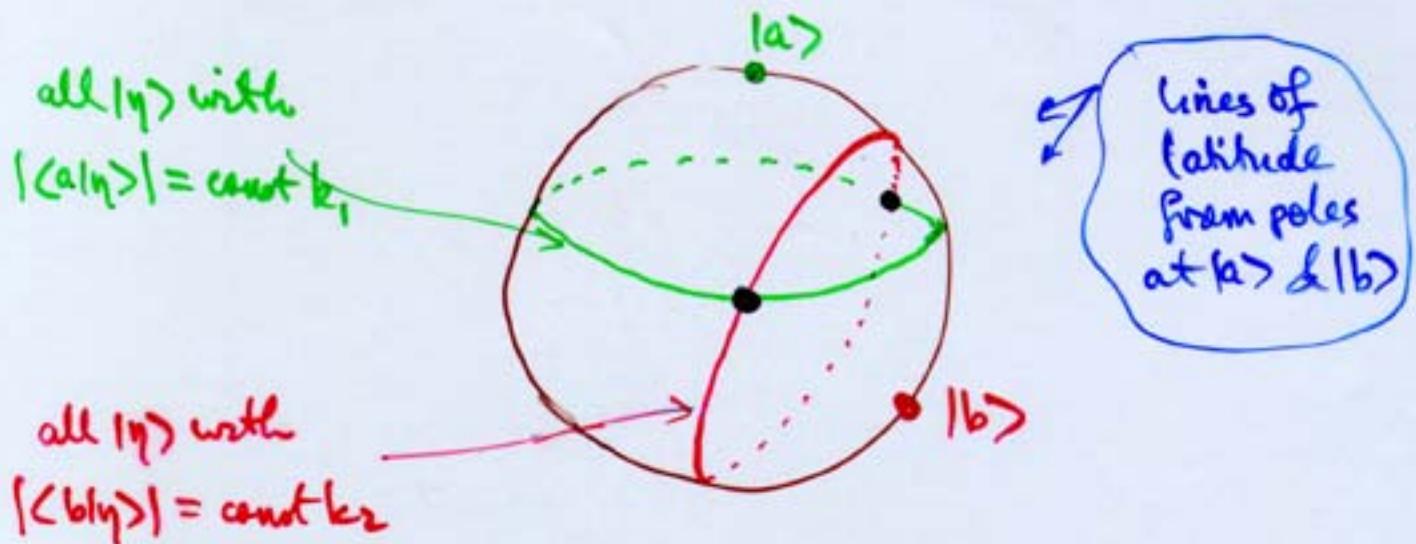
Given rays $|0\rangle, |1\rangle \Rightarrow$ ray of $x_0|0\rangle + x_1|1\rangle$ has unique meaning!

But given two nonorthogonal rays

$$|a\rangle = a_0|0\rangle + a_1|1\rangle \quad \text{rays up to overall phase}$$
$$|b\rangle = b_0|0\rangle + b_1|1\rangle$$

Where can the ray $|\xi\rangle = x_0|0\rangle + x_1|1\rangle$ (upto phase) be on Bloch sphere?

* $|\langle \xi | a \rangle|$ & $|\langle \xi | b \rangle|$ uniquely determined by rays.



If $|a\rangle \perp |b\rangle$ (antipodal) need $k_1^2 + k_2^2 = 1$ and red & green circles coincide!

- a special property of orthogonal states
(cf. cloning)

The protocol for synchronisation/syntonisation:

Starting assumptions :

- A & B separated, relatively at rest
 - they share an ensemble of (labelled) EPR states
- $$|epr\rangle = \frac{1}{\sqrt{2}} (|0\rangle_A |1\rangle_B - |1\rangle_B |0\rangle_A)$$

*Later will need this
for two values
 $\omega = \omega_1 \& \omega_2$*

Have $|epr\rangle = \frac{1}{\sqrt{2}} (|p\rangle |n\rangle - |n\rangle |p\rangle)$ for any choice
of $|p\rangle, |n\rangle$

a stationary state if A & B's particles have
some hamiltonian.

At some time (call it $t=0$):

Alice simultaneously measures all her particles in $|p\rangle/|n\rangle$ basis

- all pairs 'collapse' at $t=c$ into

$$|p\rangle_A |n\rangle_B \text{ or } |n\rangle_A |p\rangle_B \quad (\text{about 50/50})$$

and all particles begin to evolve synchronously
starting at $t=c$

Alice classically communicates her measurement results
to Bob

- so Bob can select out a $|p\rangle_B$ ensemble
(where Alice got $|n\rangle$)

So Alice and Bob now have ensembles

$$|p(t)\rangle_A \quad \text{and} \quad |p(t)\rangle_B$$

$$|p(t)\rangle = \frac{1}{\sqrt{2}} (\cos \frac{\omega t}{2} |p\rangle + \sin \frac{\omega t}{2} |n\rangle)$$

which are evolving in perfect phase for A & B.

To read the oscillations ("ticking of clocks"),
need to monitor p/n measurement results.

A can do this, but

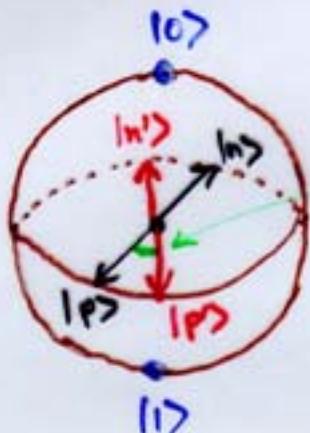
B does not know correct choice of $|p\rangle$ direction!

So:

B just chooses his own x direction
 \hat{x}_B perpendicular to \hat{z}

in language of
 spin in
 \hat{z} -magnetic field

i.e. makes his own choice of $|p'\rangle$ $|n'\rangle$ states
 on the Bloch sphere



angle S apart.

at some fixed (unknown to A&B) angle S to $|p\rangle$:

$$|p'\rangle = \frac{1}{\sqrt{2}} (e^{-iS/2} |0\rangle + e^{iS/2} |1\rangle)$$

$$|n'\rangle = \frac{1}{\sqrt{2}} (e^{-iS/2} |0\rangle - e^{iS/2} |1\rangle)$$

$$\begin{aligned} \text{so } |p(t)\rangle &= \frac{1}{\sqrt{2}} (e^{-i\omega t/2} |0\rangle + e^{i\omega t/2} |1\rangle) \\ &= \cos \frac{\omega t}{2} |p\rangle + \sin \frac{\omega t}{2} |n\rangle \\ &= \cos \frac{\omega t - S}{2} |p'\rangle + \sin \frac{\omega t - S}{2} |n'\rangle \end{aligned}$$

$$\text{Alice: prob}(p) = \frac{1}{2} (1 + \cos \omega t)$$

$$\text{Bob: prob}(p') = \frac{1}{2} (1 + \cos(\omega t - S))$$

i.e. Bob's oscillations are shifted by (unknown) S
 from Alice's.

But

δ same for both $\omega = \omega_1$ & $\omega = \omega_2$

(same choice of x_A & x_B directions for both)

Hence instead of monitoring direct oscillations,
monitor instead the beats:

$$\begin{aligned} \text{prob}_{\omega_1}(p') - \text{prob}_{\omega_2}(p') &= \frac{1}{2} [\cos(\omega_1 t - \delta) - \cos(\omega_2 t - \delta)] \\ &= -\sin\left(\frac{\omega_1 - \omega_2}{2}t\right) \cdot \sin\left(\frac{(\omega_1 + \omega_2)}{2}t + \delta\right) \end{aligned}$$

\uparrow slow beat envelope

Envelope oscillates independent of δ
starting synchronously at $t=0$ for both A & B.

Hence A & B have synchronisation.

To get a common origin of time too :

choose $\omega_1 - \omega_2$ small enough so that the
whole protocol is complete in less than
 $\frac{1}{4}$ beat period time

so first max of envelope gives a common origin.

Advantages? (assuming entanglement is in place)

- Alice & Bob need not know their relative locations
- No timing information sent in the protocol
i.e., no need to accurately measure time of arrival
 - changing properties of intervening medium have no effect.
- Synchronisation does not exist until the time that the protocol is carried out (c.f. SCT)

[c.f. entanglement based QKD vs BB84]

Main (theoretical) problem:

How to establish the entanglement?

e.g. is it any "easier" than just distributing accurately ticking clocks?

e.g. entanglement purification techniques implicitly assume time synchrony of local operations?